

A wave-function model for the CP -violation in mesons

M. Courbage¹, T. Durt² and S.M. Saberi Fathi³

Abstract

In this paper we propose to associate a temporal two-component wave-function to the decay process of meson particles. This simple quantum model provides a good estimation of the CP symmetry violation parameter. This result is based on our previous paper [1] where we have shown that the two-level Friedrichs Hamiltonian model makes it possible to provide a qualitatively correct phenomenological model of kaons physics. In this previous paper, we derived a violation parameter that is 14 times larger than the measured quantity. In the present paper we improve our estimation of the violation and obtain the right order of magnitude. The improvement results from a renormalized superposition of the probability amplitudes describing short and long exponential decays. The renormalization occurs because the amplitudes that we are dealing with are associated to the decay rate, and not to the integrated decay rate or survival probability as is usually the case in standard approaches to CP -violation. We also compare with recent experimental data for the mesons D and B and also there the agreement between our model and experimental data is quite satisfying.

1 Introduction

The status of time in the quantum theory is still a controversial subject [2]. For instance, although standard quantum mechanics allows us in principle to bring an unambiguous answer to the question *is a certain particle located inside a given space region at time t ?* (the answer is: the probability that the answer is Yes is equal to the integral of the modulus square of the wave function, at time t , over that region), it does not allow to answer to the question *at what time will the particle enter this region?* One could equivalently formulate this question in the form *what is the probability that a particle hits a screen during a given time interval?* Or *at what time will the particle hit the screen?*, which justifies why the problem of deriving a temporal distribution from Schrödinger's wave function (instead of a spatial distribution) is often referred to in the literature as the so-called *screen problem* [3, 4] or arrival-time problem. One could believe that the source of the confusion is that standard quantum mechanics is not a

¹Laboratoire Matière et Systèmes Complexes (MSC), UMR 7057 CNRS et Université Paris 7- Denis Diderot, Case 7056, Bâtiment Condorcet, 10, rue Alice Domon et Lonie Duquet 75205 Paris Cedex 13, FRANCE. **email:** maurice.courbage@univ-paris-diderot.fr

²TENA-TONA Vrije Universiteit Brussel, Pleinlaan 2, B-1050 Brussels, Belgium. **email:** thomdurt@vub.ac.be

³Laboratoire de Physique Théorique et Modélisation (LPTM), UMR 8089, CNRS et Université de Cergy-Pontoise, Site Saint-Matrin, 2, rue Adolphe Chauvin, 95302 Cergy-Pontoise Cedex, FRANCE. **email:** majid.saberi@cergy.fr

relativistic theory and that in the framework of relativistic quantum field theory these questions possess an unambiguous answer but this is not the case. Even localization in space becomes problematic in quantum field theory [5].

The screen problem is close to the tunneling phenomenon [6] which in turn presents many analogies with the decay process [7]; one could indeed modellize the decay of an unstable state or resonance as a tunneling through the barrier of potential that prevents instantaneous decay. Then, if we can predict at which time the particle will tunnel, we can also predict the distribution of decay times.

Although there is no unambiguous recipe for deriving temporal probability distributions in the quantum theory, in the majority of standard approaches to the decay process (among which the celebrated Wigner-Weisskopf approach), metastable states or resonances are formally characterized by a complex energy, of which the real part is proportional to the mass of the particle and of which the imaginary part is proportional to the inverse of the lifetime of the resonance. For such exponential decay laws, the “*integrated survival probability*” $P_s(t)$, from an initial time 0 up to t , obeys

$$P_s(t) = \frac{\psi^*(t)\psi(t)}{\psi^*(0)\psi(0)}. \quad (1)$$

Indeed, setting

$$\psi(t) = \psi(0)e^{-i(m-\frac{i}{2}\Gamma)t}, \quad (2)$$

we get [8], as a consequence of equation (1),

$$P_s(t) = e^{-\frac{t}{\tau}} \quad (3)$$

where the lifetime τ is equal to the inverse of the so-called decay rate Γ : $\Gamma = \frac{1}{\tau}$. These exponential amplitudes are commonly used in particle physics.

It is well-known actually that this exponential decay-law is a mere approximation and that one should expect discrepancies from it in the short-time (Zeno regime) [4] and long-time behavior of unstable quantum systems. Nevertheless, it is not easy to observe experimentally such discrepancies [9] because they mostly affect non-significant parts of the statistical data that can be collected about the distribution of decay-times and we shall not consider these extreme cases in the rest of the paper. Experimentally, it is most often easier to measure, instead of the *integrated survival probability*, the (temporal) “*density of probability*” of decay $p_d(t)$ or *decay rate* which is equal to minus the time derivative of $P_s(t)$: $p_d(t) = d(1 - P_s(t))/dt = -dP_s(t)/dt$. In the case of an exponential decay nevertheless this subtle point is often overlooked without prejudicial consequences because $P_s(t)$ is *proportional* to $p_d(t)$.

Indeed, deriving both sides of the expression (3) relatively to time, we obtain

$$p_d(t) = -\frac{dP_s(t)}{dt} = \Gamma e^{-\Gamma t} = p_d(0) e^{-\Gamma t}. \quad (4)$$

One can also, still in the case of exponential decays, express the decay rate in function of the norm of the state as follows

$$p_d(t) = -\frac{dP_s(t)}{dt} = p_d(0) \frac{\psi^*(t)\psi(t)}{\psi^*(0)\psi(0)}. \quad (5)$$

If one considers more complex behaviors, such as the superposition of two exponential decay processes with different lifetimes, then $P_s(t)$ is *no longer proportional* to $p_d(t)$! This is for instance what occurs in experimental observations of CP -violation during which populations of pairs of pions are estimated along the decay of unstable short lived and long lived kaons, and where an interference effect is exhibited leading to CP -violation.

In the present paper we shall show that, if one makes a correct superposition of the amplitude associated to the densities of the short and long decay processes, the CP -violation parameter that we derive must be re-estimated. We show how a simple model that we developed in the past provides a correct prediction of the magnitude of the observed CP -violation. We compare with more recent experimental of the mesons D and B.

2 Phenomenology of kaon particles

Let us first recall some basic experimental facts. Kaons are bosons that were discovered in the forties during the study of cosmic rays. They are produced by collision processes in nuclear reactions during which the strong interactions dominate. They appear in pairs K^0, \bar{K}^0 [10, 11].

The K mesons are eigenstates of the parity operator P : $P|K^0\rangle = -|K^0\rangle$, and $P|\bar{K}^0\rangle = -|\bar{K}^0\rangle$. K^0 and \bar{K}^0 are charge conjugate to each other $C|K^0\rangle = |\bar{K}^0\rangle$, and $C|\bar{K}^0\rangle = |K^0\rangle$. We get thus,

$$CP|K^0\rangle = -|\bar{K}^0\rangle, \quad CP|\bar{K}^0\rangle = -|K^0\rangle. \quad (6)$$

Clearly $|K^0\rangle$ and $|\bar{K}^0\rangle$ are not CP -eigenstates, but the following combinations

$$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle), \quad |K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle),$$

are CP -eigenstates:

$$CP|K_1\rangle = +|K_1\rangle, \quad CP|K_2\rangle = -|K_2\rangle. \quad (7)$$

In the absence of matter, kaons disintegrate through weak interactions [11]. Actually, K^0 and \bar{K}^0 are distinguished by their mode of *production*. K_1 and K_2 are the decay modes of kaons. In absence of CP -violation, the weak disintegration process distinguishes the K_1 states which decay only into “ 2π ” while the K_2 states decay into “ $3\pi, \pi e \nu, \dots$ ” [12]. The lifetime of the K_1 kaon is short ($\tau_S \approx 8.92 \times 10^{-11}$ s), while the lifetime of the K_2 kaon is quite longer ($\tau_L \approx 5.17 \times 10^{-8}$ s) [10].

CP -violation was discovered by Christenson et al. [13]. CP -violation means that the long-lived kaon can also decay to “ 2π ”. Then, the CP symmetry is slightly violated (by a factor of 10^{-3}) by weak interactions so that the CP eigenstates K_1 and K_2 are not exact eigenstates of the decay interaction. Those exact states are characterized by lifetimes that are in a ratio of the order of 10^{-3} , so that they are called the short-lived state (K_S) and long-lived state (K_L). They can be expressed as coherent superpositions of the K_1 and K_2 eigenstates through

$$\begin{aligned} |K_L\rangle &= \frac{1}{\sqrt{1+|\epsilon|^2}} [\epsilon |K_1\rangle + |K_2\rangle], \\ |K_S\rangle &= \frac{1}{\sqrt{1+|\epsilon|^2}} [|K_1\rangle + \epsilon |K_2\rangle], \end{aligned} \quad (8)$$

where ϵ is a complex CP -violation parameter, $|\epsilon| \ll 1$. K_L and K_S are the eigenstates of the Hamiltonian for the mass-decay matrix [11, 12] which has the following form in the basis $|K^0\rangle$ and $|\bar{K}^0\rangle$ has the following form:

$$H = M - \frac{i}{2}\Gamma \equiv \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{21} - \frac{i}{2}\Gamma_{21} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix} \quad (9)$$

where M and Γ are individually Hermitian since they correspond to observables (mass and lifetime). The corresponding eigenvalues of the mass-decay matrix are equal to

$$E_L = m_L - \frac{i}{2}\Gamma_L, \quad \text{and} \quad E_S = m_S - \frac{i}{2}\Gamma_S. \quad (10)$$

We shall derive an explicit expression of this mass-decay matrix in our model.

The CP -violation was established by the observation that K_L decays not only via three-pion, which has natural CP parity, but also via the two-pion (“ 2π ”) mode with an experimentally observed violation amplitude of the order of 10^{-3} .

Let us now reconsider how the simple model (8) and (10) is related to the experimental data. A series of detections is performed at various distances from the source of a neutral kaon beam in order to estimate the variation of the populations of emitted pion π^+, π^- pairs in function of the proper time. The initial state is a neutral kaon state

$$|K^0\rangle = \frac{1}{\sqrt{2}}(|K_1\rangle + |K_2\rangle) = \frac{\sqrt{1+|\epsilon|^2}}{\sqrt{2}(1+\epsilon)}(|K_S\rangle + |K_L\rangle). \quad (11)$$

As the π^+, π^- pairs are CP -eigenstates for the eigenvalue $+1$, their population is proportional to the K_1 population. In the case that ϵ equals 0 (no CP -violation), $|K_S\rangle = |K_1\rangle$ and $|K_L\rangle = |K_2\rangle$ so that, for times quite longer than τ_S , no π^+, π^- pair is likely to be observed. The experiment shows on the contrary that these pairs are observed.

2.1 Standard formulation of CP -violation

The standard modeling of the process goes as follows: in accordance with the expression (11), the linearity of Schrödinger equation, the transition amplitude $\psi_1(t)$ towards the K_1 state at time t obeys

$$\psi_1(t) = \langle K_1 | K^0(t) \rangle = \frac{\sqrt{1+|\epsilon|^2}}{\sqrt{2}(1+\epsilon)} (\langle K_1 | K_S(t) \rangle + \langle K_1 | K_L(t) \rangle) \quad (12)$$

and by using

$$|K_S(t)\rangle = |K_S\rangle e^{-iE_S t}, \quad |K_L(t)\rangle = |K_L\rangle e^{-iE_L t} \quad (13)$$

we obtain

$$\psi_1(t) = \frac{\sqrt{1+|\epsilon|^2}}{\sqrt{2}(1+\epsilon)} (\langle K_1 | K_S \rangle e^{-iE_S t} + \langle K_1 | K_L \rangle e^{-iE_L t}). \quad (14)$$

So that, combining with equation (8) we get

$$\psi_1(t) = \frac{1}{\sqrt{2}(1+\epsilon)} \left(e^{-i(m_S - \frac{i}{2}\Gamma_S)t} + \epsilon e^{-i(m_L - \frac{i}{2}\Gamma_L)t} \right) \quad (15)$$

Now, some confusion exists in the literature regarding how to proceed next. We tried to reconstitute a derivation of the generic formula that we shall present here, having in mind that this derivation presents certain weaknesses (that we shall address in a separate publication), but, as we propose in a next section an alternative approach to the problem, this does serve our purposes.

It is generally admitted [10, 11] that the intensity $I(t)$ of π^+, π^- pair detection at time t obeys the constraint

$$I(t) = |\psi_1(t)|^2. \quad (16)$$

Then we get by a straightforward computation

$$I(t) = \frac{I(t=0)}{|1+\epsilon|^2} \left(e^{-\Gamma_S t} + |\epsilon|^2 e^{-\Gamma_L t} + |\epsilon| e^{-(\frac{\Gamma_S + \Gamma_L}{2})t} \cos(\Delta m t + \arg(\epsilon)) \right)$$

where I_0 is a global factor constant in time, and $\Delta m = |m_L - m_S|$, so that an interference term is likely to appear, that reveals the existence of a CP -violation. The existence of such an effect was confirmed by experiments [13]. By fitting this oscillating contribution with the observed data one derives an estimation of the mass difference between the short and long lived state as well as the phase of ϵ and its amplitude. All this leads to an experimental estimation of ϵ (that we shall denote ϵ^{exp}) [14]

$$|\epsilon^{\text{exp}}| = (2.232 \pm 0.007) \times 10^{-3}, \quad \arg(\epsilon^{\text{exp}}) = (43.5 \pm 0.7)^\circ. \quad (17)$$

2.2 Alternative estimation of $|\epsilon|$

There exist different experimental quantities that depend on the CP violation parameter. This means that there are alternative experiments that make it possible to estimate the value of ϵ^{exp} . An alternative way to estimate the modulus of ϵ^{exp} consists of measuring ratios of production rates. For instance, the ratio between the production rate of charged pion pairs emitted by Long-lived states and the one corresponding to Short-lived states obeys

$$\frac{\text{Proba. per unit of time } (K_L \rightarrow \pi^+, \pi^-)}{\text{Proba. per unit of time } (K_S \rightarrow \pi^+, \pi^-)} = \frac{|\text{Amplitude } (K_L \rightarrow \pi^+, \pi^-)|^2}{|\text{Amplitude } (K_S \rightarrow \pi^+, \pi^-)|^2}. \quad (18)$$

Now,

$$\begin{aligned} \epsilon &= \frac{\text{Amplitude } (K_L \rightarrow K_1)}{\text{Amplitude } (K_S \rightarrow K_1)} = \frac{\text{Amplitude } (K_L \rightarrow K_1) \times \text{Amplitude } (K_1 \rightarrow \pi^+, \pi^-)}{\text{Amplitude } (K_S \rightarrow K_1) \times \text{Amplitude } (K_1 \rightarrow \pi^+, \pi^-)} \\ &= \frac{\text{Amplitude } (K_L \rightarrow \pi^+, \pi^-)}{\text{Amplitude } (K_S \rightarrow \pi^+, \pi^-)}, \end{aligned} \quad (19)$$

(where we made use of the fact that π^+, π^- are $CP=+1$ eigenstates like K_1 and belong to a space orthogonal to the $CP=-1$ eigenspace to which K_2 states belong), so that

$$\frac{\text{Proba. per unit of time } (K_L \rightarrow \pi^+, \pi^-)}{\text{Proba. per unit of time } (K_S \rightarrow \pi^+, \pi^-)} = |\epsilon|^2 \quad (20)$$

The estimation of $|\epsilon|$ that is obtained by measuring this ratio coincides with the value $|\epsilon^{\text{exp}}|$ mentioned above, which is also an indirect proof of the relevance and of the consistency of the standard modeling of CP -violation.

3 Theoretical model

In 1957, Lee, Oehme and Yang (LOY) [15] derived a time evolution equation of the neutral kaons using the Weisskopf-Wigner approach to the decay of quantum systems [16]. LOY's Hamiltonian describes (K^0, \bar{K}^0) evolution modes. Later on, the LOY equation has been improved by several authors [17, 18, 19, 20, 21] in order to obtain a correction due to the departure from the pure exponential decay for short and long times. Chiu and Sudarshan obtained a numerical estimate of the modulus of the CP -violation parameter that is 30 times the experimental value. Our approach, based on the derivation of a master equation from a Friedrichs Hamiltonian [22, 23] in terms of the decaying modes (K_1, K_2) under weak coupling approximation, provided a new estimate of the modulus of the CP -violation parameter that is at first sight 14 times greater than the experimental modulus while the estimated phase is roughly correct.

The two-level Friedrichs interaction Hamiltonian couples two modes and a continuous degree of freedom in such a way that the Schrödinger equation is [22, 24, 25]

$$\begin{pmatrix} \omega_1 & 0 & \lambda_1^* \\ 0 & \omega_2 & \lambda_2^* \\ \lambda_1 & \lambda_2 & \omega \end{pmatrix} \begin{pmatrix} f_1(t) \\ f_2(t) \\ g(\omega, t) \end{pmatrix} = i \frac{\partial}{\partial t} \begin{pmatrix} f_1(t) \\ f_2(t) \\ g(\omega, t) \end{pmatrix}. \quad (21)$$

In this model, the energies ω of the different modes of the continuum range from $-\infty$ to $+\infty$. The masses $\omega_{1(2)}$ represent the energies of the discrete levels, and the factors $\lambda_{1(2)}$ represent the couplings to the continuum of decay product. Consequently, the amplitudes of the discrete and continuous modes obey

$$\omega_1 f_1(t) + \lambda_1^* \int_{-\infty}^{\infty} d\omega g(\omega, t) = i \frac{\partial f_1(t)}{\partial t}, \quad (22)$$

$$\omega_2 f_2(t) + \lambda_2^* \int_{-\infty}^{\infty} d\omega g(\omega, t) = i \frac{\partial f_2(t)}{\partial t}, \quad (23)$$

$$\lambda_1 f_1(t) + \lambda_2 f_2(t) + \omega g(\omega, t) = i \frac{\partial g(\omega, t)}{\partial t}. \quad (24)$$

Integrating the last equation we obtain $g(\omega, t)$ assuming $g(\omega, t=0) = 0$:

$$g(\omega, t) = -i e^{-i\omega t} \int_0^t d\tau [\lambda_1 f_1(\tau) + \lambda_2 f_2(\tau)] v(\omega) e^{i\omega\tau}, \quad (25)$$

then, we substitute $g(\omega, t)$ in the above equation (22) we obtain

$$i \frac{\partial f_1(t)}{\partial t} = \omega_1 f_1(t) - i \lambda_1^* \int d\omega |v(\omega)|^2 e^{-i\omega t} \int_0^t d\tau [\lambda_1 f_1(\tau) + \lambda_2 f_2(\tau)] e^{i\omega\tau}, \quad (26)$$

we also can obtain the same relation for $f_2(t)$ as

$$i \frac{\partial f_2(t)}{\partial t} = \omega_2 f_2(t) - i \lambda_2^* \int d\omega |v(\omega)|^2 e^{-i\omega t} \int_0^t d\tau [\lambda_1 f_1(\tau) + \lambda_2 f_2(\tau)] e^{i\omega\tau}. \quad (27)$$

One can obtain [1]

$$i \frac{\partial}{\partial t} \begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix} = \begin{pmatrix} \omega_1 - i\pi |\lambda_1|^2 & -i\pi \lambda_1 \lambda_2^* \\ -i\pi \lambda_1^* \lambda_2 & \omega_2 - i\pi |\lambda_2|^2 \end{pmatrix} \begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix}. \quad (28)$$

Thus, we obtain an effective non-Hermitian Hamiltonian evolution, $H_{\text{eff}} = M - i\frac{\Gamma}{2}$. The eigenvalues of the above effective Hamiltonian under the weak coupling constant approximation become:

$$\omega_+ = \omega_1 - i\pi|\lambda_1|^2 + O(\lambda^4), \quad \omega_- = \omega_2 - i\pi|\lambda_2|^2 + O(\lambda^4), \quad (29)$$

In a first and very rough approximation, the eigenvectors of the effective Hamiltonian are the same as the postulated kaons states.

$$|f_+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |K_1\rangle \quad \text{and} \quad |f_-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |K_2\rangle. \quad (30)$$

Phenomenology imposes that the complex Friedrichs energies ω_{\pm} coincide with the observed complex energies. The Friedrichs energies depend on the choice of the four parameters ω_1 , ω_2 , λ_1 and λ_2 and the observed complex energies are directly derived from the experimental determination of four other parameters, the masses m_S and m_L and the lifetimes τ_S and τ_L . We must thus adjust the theoretical parameters in order that they fit the experimental data. This can be done by comparing the eigenvalue of the effective matrix with the eigenvalue of the mass-decay matrix which is taken in the equation (10). Finally, we have

$$\begin{aligned} \omega_1 &= m_S, & 2\pi|\lambda_1|^2 &= \Gamma_S, \\ \omega_2 &= m_L, & 2\pi|\lambda_2|^2 &= \Gamma_L. \end{aligned} \quad (31)$$

The above identities yields

$$\lambda_1 = \sqrt{\frac{\Gamma_S}{2\pi}} e^{i\theta_S}, \quad \lambda_2 = \sqrt{\frac{\Gamma_L}{2\pi}} e^{i\theta_L} \quad (32)$$

where θ_S and θ_L are real constants.

3.1 *CPT* invariance

Let us now discuss the *CPT* invariance in our model. As mentioned in the texts books like [11, 12], *CPT* invariance imposes some conditions on the mass-decay matrix, i.e.

$$M_{11} = M_{22}, \quad \Gamma_{11} = \Gamma_{22}, \quad M_{12} = M_{21}^* \quad \text{and} \quad \Gamma_{12} = \Gamma_{21}^* \quad (33)$$

in the K^0 and \bar{K}^0 bases. But, we note that our effective Hamiltonian is written in the K_1 and K_2 bases. Thus, we have to rewrite in the K^0 and \bar{K}^0 bases. Thus, the transformation matrix T from the K_1 and K_2 bases to the K^0 and \bar{K}^0 bases is obtained as

$$T = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = T^{-1}. \quad (34)$$

Then, the effective Hamiltonian in the K^0 and \bar{K}^0 bases, H_{eff}^{00} is obtained by

$$H_{\text{eff}}^{00} = T H_{\text{eff}} T^{-1} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \omega_1 - i\pi|\lambda_1|^2 & -i\pi\lambda_1\lambda_2^* \\ -i\pi\lambda_1^*\lambda_2 & \omega_2 - i\pi|\lambda_2|^2 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \quad (35)$$

we have, $H_{\text{eff}}^{00} =$

$$\begin{pmatrix} (m_S + m_L) - \frac{i}{2}(\Gamma_S + \Gamma_L + 2\sqrt{\Gamma_S \Gamma_L} \cos \Delta\theta), & (m_S - m_L) - \frac{i}{2}(\Gamma_S - \Gamma_L + 2i\sqrt{\Gamma_S \Gamma_L} \sin \Delta\theta) \\ (m_S - m_L) - \frac{i}{2}(\Gamma_S - \Gamma_L - 2i\sqrt{\Gamma_S \Gamma_L} \sin \Delta\theta), & (m_S + m_L) - \frac{i}{2}(\Gamma_S + \Gamma_L - 2\sqrt{\Gamma_S \Gamma_L} \cos \Delta\theta) \end{pmatrix}. \quad (36)$$

where $\Delta\theta = \theta_S - \theta_L$. *CPT* invariance conditions in (33) impose that

$$\Delta\theta = k\pi + \frac{\pi}{2}, \quad (k = \dots, -1, 0, 1, \dots). \quad (37)$$

Here we choose $k = -1$, consequently, $\Delta\theta = -\frac{\pi}{2}$. Then, we have

$$\begin{aligned} M_{11} &= M_{22} = (m_S + m_L), & \Gamma_{11} &= \Gamma_{22} = \Gamma_S + \Gamma_L, \\ M_{12} &= M_{21}^* = (m_S - m_L), & \Gamma_{12} &= \Gamma_{21}^* = \Gamma_S - \Gamma_L - 2i\sqrt{\Gamma_S \Gamma_L}. \end{aligned} \quad (38)$$

3.2 *CP*-violation

Let us study in this case the *CP*-violation. The Friedrichs model allows us to estimate the value of ϵ . For this purpose, the effective Hamiltonian (28) acts on the $|K_S\rangle$ vector states (8) as an eigenstate corresponding to the eigenvalue $\omega_+ = \omega_1 - i\pi\lambda_1^2 = m_S - i\frac{\Gamma_S}{2}$, so that we must impose that $H_{\text{eff}}^{(1)}(\epsilon) = \omega_+^{(1)}$, from which we obtain after straightforward calculations that

$$\epsilon = \frac{-i\pi\lambda_1^*\lambda_2}{(\omega_2 - \omega_1) - i\pi(|\lambda_2|^2 - |\lambda_1|^2)} \quad (39)$$

and if we replace λ 's and ω 's by corresponding values in equation (31) we have,

$$\epsilon = \frac{-\frac{i}{2}\sqrt{\Gamma_L \Gamma_S} e^{i\frac{\pi}{2}}}{(m_L - m_S) - \frac{i}{2}(\Gamma_L - \Gamma_S)} = \frac{\frac{1}{2}\sqrt{\Gamma_L \Gamma_S}}{(m_L - m_S) - \frac{i}{2}(\Gamma_L - \Gamma_S)}. \quad (40)$$

Similarly, the effective Hamiltonian (28) acts on the $|K_L\rangle$ vector states (8) as an eigenstate corresponding to the eigenvalue $\omega_- = \omega_2 - i\pi\lambda_2^2 = m_L - i\frac{\Gamma_L}{2}$, so that we must impose that $H_{\text{eff}}^{(\epsilon)}(\epsilon) = \omega_-^{(\epsilon)}$, from which we obtain after straightforward calculations that

$$\epsilon = \frac{i\pi\lambda_1\lambda_2^*}{(\omega_2 - \omega_1) - i\pi(|\lambda_2|^2 - |\lambda_1|^2)} \quad (41)$$

and if we replace λ 's and ω 's by corresponding values in equation (31) we have,

$$\epsilon = \frac{\frac{i}{2}\sqrt{\Gamma_L \Gamma_S} e^{-i\frac{\pi}{2}}}{(m_L - m_S) - \frac{i}{2}(\Gamma_L - \Gamma_S)} = \frac{\frac{1}{2}\sqrt{\Gamma_L \Gamma_S}}{(m_L - m_S) - \frac{i}{2}(\Gamma_L - \Gamma_S)}. \quad (42)$$

K_S and K_L provide the same expression for ϵ as can be seen from the equations (40) and (42).

At this level we introduce a fundamentally new ingredient that constitutes a breakthrough relatively to standard text-book approaches. This ingredient consists of associating a temporal two-component wave-function to the decay rate. It is based on an analogy with spin 1/2 spatial wave-functions that we present now.

3.3 The spin 1/2 analogy

Let us consider the two-components of the Pauli wave function $(\Psi_1(x, T), \Psi_2(x, T))$ associated to a spinor at time T . The probability to find spin “up” $((1,0))$ (spin “down” $((0,1))$) at time T in the interval $[x, x + dx]$ is, according to the usual rules of Quantum Mechanics, equal to dx times $|\Psi_1(x, T)|^2$ ($|\Psi_2(x, T)|^2$).

Let us now replace space by time, the spin operator by the CP pseudo-spin operator, and the measurement of the position of a particle by the measurement of the time at which decay occurs. The modulus square of the first component of the wave function is then equal to the decay rate in the $CP = +1$ sector. An initial K_0 state is seen to correspond, in virtue of this analogy, to a fifty-fifty coherent superposition state of a Short state $\frac{1}{\sqrt{1+|\epsilon|^2}} \binom{1}{\epsilon}$ and of a Long state $\frac{1}{\sqrt{1+|\epsilon|^2}} \binom{\epsilon}{1}$. The $CP = +1$ and -1 components of the temporal wave function $(\tilde{\psi}_1(t), \tilde{\psi}_2(t))$ associated to this state are thus equal to

$$\tilde{\psi}_1(t) = \frac{1}{\sqrt{2}\tilde{N}} \left(\sqrt{\Gamma_S} e^{-i(m_S - \frac{1}{2}\Gamma_S)t} + \epsilon \sqrt{\Gamma_L} e^{-i(m_L - \frac{1}{2}\Gamma_L)t} \right) \quad (43)$$

and

$$\tilde{\psi}_2(t) = \frac{1}{\sqrt{2}\tilde{N}} \left(\epsilon \sqrt{\Gamma_S} e^{-i(m_S - \frac{1}{2}\Gamma_S)t} + \sqrt{\Gamma_L} e^{-i(m_L - \frac{1}{2}\Gamma_L)t} \right). \quad (44)$$

The probability to find a $CP = +1$ kaon (or K_1 particle) in the temporal interval $[t, t + dt]$ is, according to the usual rules of quantum mechanics, equal to:

$$|\tilde{\psi}_1(t)|^2 = \frac{1}{2\tilde{N}^2} \left(\Gamma_S e^{-\Gamma_S t} + |\epsilon|^2 \Gamma_L e^{-\Gamma_L t} + 2 \operatorname{Re} \left(\epsilon \sqrt{\Gamma_S \Gamma_L} e^{-i(m_L - m_S + \frac{\Gamma_S + \Gamma_L}{2})t} \right) \right) dt. \quad (45)$$

Similarly, the probability to find a $CP = -1$ kaon (or K_2 particle) in the interval $[t, t + dt]$ is equal to:

$$|\tilde{\psi}_2(t)|^2 = \frac{1}{2\tilde{N}^2} \left(|\epsilon|^2 \Gamma_S e^{-\Gamma_S t} + \Gamma_L e^{-\Gamma_L t} + 2 \operatorname{Re} \left(\epsilon \sqrt{\Gamma_S \Gamma_L} e^{-i(m_S - m_L + \frac{\Gamma_S + \Gamma_L}{2})t} \right) \right) dt. \quad (46)$$

Normalization is imposed by the requirement that (i) at time 0, when a particle is prepared, its survival probability is equal to 1, (ii) the survival probability tends to 0 when time tends to infinity, which means that $\int_0^\infty dt (-) dP_s(t)/dt = \int_0^\infty dt p_d(t) = 1$, and (iii) $p_d(t) = \psi^*(t)\psi(t)$, in analogy with the normalization condition that is imposed to spatially extended wave functions in first quantization procedure. \tilde{N} is thus chosen in order to normalize the probability of decay to 1: $\int_0^\infty (|\tilde{\psi}_1(t)|^2 + |\tilde{\psi}_2(t)|^2) dt = 1$. This means that

$$\tilde{N}^2 = 1 + |\epsilon|^2 + \left(\frac{\sqrt{\Gamma_S \Gamma_L} (\Gamma_S + \Gamma_L)}{(\Delta m)^2 + (\frac{\Gamma_S + \Gamma_L}{2})^2} \right) \operatorname{Re}(\epsilon). \quad (47)$$

3.4 Renormalization of the violation parameter

Let us now reconsider CP -violation, having in mind the wave-function model described in the previous section. In the case that Short and Long decay processes coherently interfere, our

normalization criterion imposes that the effective transition amplitude $\tilde{\psi}_1(t)$ towards the K_1 state at time t , is equal to

$$\begin{aligned}\tilde{\psi}_1 &= \frac{1}{\tilde{N}} \left(\langle K_1 | K_S \rangle \sqrt{-2\text{Im}(E_S)} e^{-iE_S t} + \langle K_1 | K_L \rangle \sqrt{-2\text{Im}(E_L)} e^{-iE_L t} \right) \\ &= \frac{1}{\tilde{N}} \left(\sqrt{\Gamma_S} e^{-iE_S t} + \epsilon \sqrt{\Gamma_L} e^{-iE_L t} \right)\end{aligned}\quad (48)$$

where \tilde{N} obeys (47). Because of our new choice of normalization, there appear new normalization factors $\sqrt{-2\text{Im}(E_{L(S)})}$ that were not present at the level of equation (14). Thus, the theoretically estimated intensity $I(t)$ has now the following form:

$$\begin{aligned}I(t) &= \frac{I(t=0)}{|1 + \epsilon^{\text{th}}|^2} \left(e^{-2\pi\lambda_1^2 t} + |\epsilon^{\text{th}}|^2 e^{-2\pi\lambda_2^2 t} + 2|\epsilon^{\text{th}}| e^{-\pi(\lambda_1^2 + \lambda_2^2)t} \cos(\Delta\omega t + \arg(\epsilon^{\text{th}})) \right) \\ &= \frac{I(t=0)}{|1 + \epsilon^{\text{th}}|^2} \left(e^{-\Gamma_S t} + |\epsilon^{\text{th}}|^2 e^{-\Gamma_L t} + |\epsilon^{\text{th}}| e^{-(\frac{\Gamma_S + \Gamma_L}{2})t} \cos(\Delta m t + \arg(\epsilon)) \right)\end{aligned}\quad (49)$$

where we define ϵ^{th} by the renormalisation condition

$$\epsilon^{\text{th}} = \epsilon \sqrt{\frac{\Gamma_L}{\Gamma_S}}, \quad (50)$$

so that

$$\epsilon^{\text{th}} = \epsilon \sqrt{\frac{\Gamma_L}{\Gamma_S}} = \frac{\Gamma_L}{\Gamma_S} \frac{\frac{1}{2}}{\frac{\Delta m}{\Gamma_S} - i \frac{\Delta \Gamma}{2\Gamma_S}}. \quad (51)$$

4 Experimental confirmations

4.1 Kaons

By using the experimental ratio $\frac{(m_L - m_S)}{-(\Gamma_L - \Gamma_S)} \approx \Delta m \tau_S \approx 0.5$ and the above experimental values of $\Gamma_L, \Gamma_S, m_L, m_S$, we obtain the following estimated value for ϵ^{th} :

$$\epsilon^{\text{th}} = \left(\frac{1.82}{\sqrt{2}} \times 10^{-3} \right) \times e^{i(46.77)^\circ} \approx 0.6 \epsilon^{\text{exp}} \quad (52)$$

which shows that our simple model gives a rather good rough estimation of CP -violation.

Actually, the Friedrichs model still possesses adjustable quantities like the cut-off of the coupling constants $\lambda_{1,2}$. Different choices for the cut-off lead to slightly different estimations of ϵ as we have shown in Ref.[21] so that at this level we are fully satisfied if we obtain a rough agreement between our predictions for ϵ^{th} and its experimental counterpart ϵ^{exp} .

4.2 Internal consistency of the renormalization prescription for ϵ

As we mentioned in the section 2.2, there exist different experimental quantities that depend on ϵ . In order to establish the self-consistency of our approach it is important to check that these quantities are renormalized in a similar fashion.

Let us check that it is well so, repeating the reasoning of the section 2.2 in the wave-function approach. In the wave-function approach we find that the amplitude that K_L decays in the $CP = +1$ sector is weighted by a factor $\sqrt{\Gamma_L}$. Similarly, the amplitude that K_S decays in the $CP = +1$ sector must be renormalized by a factor $\sqrt{\Gamma_S}$.

$$\frac{\text{Production rate of } (\pi^+, \pi^-) \text{ from } K_L}{\text{Production rate of } (\pi^+, \pi^-) \text{ from } K_S} = \frac{\text{Proba. per unit of time } (K_L \rightarrow \pi^+, \pi^-)}{\text{Proba. per unit of time } (K_S \rightarrow \pi^+, \pi^-)} = \frac{|\epsilon|^2 \Gamma_L}{\Gamma_S} = |\epsilon^{\text{th}}|^2. \quad (53)$$

As we see from the previous equation, in the wave-function approach, the same renormalization condition (50) is consistently used that we measure CP -violation through interference effects or through ratios of production rates. This establishes the consistency of our approach.

Our model also possesses some predicting power for what concerns other particles like B and D particles, because the CP -violation parameter is related to other quantities (life times and masses) through the constraint (40) as we shall now show.

4.3 B-mesons

The other example is the CP -violation in the decay of B_s^0 and \bar{B}_s^0 . The experimental values are [26]

$$\frac{\Delta\Gamma_s}{2\Gamma_s} = 0.069_{-0.062}^{+0.058}, \quad \frac{1}{\Gamma_s} = 1.470_{-0.027}^{+0.026} \text{ ps}, \quad (54)$$

or equivalently ($\Gamma_{L,H} = \Gamma_s \pm \Delta\Gamma_s/2$),

$$\frac{1}{\Gamma_L} = 1.419_{-0.038}^{+0.039} \text{ ps}, \quad \frac{1}{\Gamma_H} = 1.525_{-0.063}^{+0.062} \text{ ps}, \quad (55)$$

and the difference of masses is

$$\Delta m = 17.7_{-2.1}^{+6.4} \text{ ps}^{-1} \quad (56)$$

and the experimental CP -violation parameter of the B meson is [26, 27]:

$$\mathcal{A}_{SL}^{\text{exp}} \simeq 4\mathcal{R}e(\epsilon_B^{\text{exp}}) = (-0.4 \pm 5.6) \times 10^{-3} \Rightarrow \left| \frac{q}{p} \right|^{\text{exp}} = 1.0002 \pm 0.0028. \quad (57)$$

where $\frac{\mathcal{A}_{SL}^{\text{exp}}}{2} \approx 1 - \left| \frac{q}{p} \right|^{\text{exp}}$. By replacing in the equation (51) we obtain:

$$\epsilon_B^{\text{th}} = \frac{\Gamma_L}{\Gamma_H} \frac{\frac{1}{2}}{\frac{\Delta m}{\Gamma_s} - i \frac{\Delta\Gamma_s}{2\Gamma_s}} = 0.018 + 0.047 \times 10^{-3} i \quad (58)$$

Thus, our theoretical $\left| \frac{q}{p} \right|^{\text{th}}$ prediction is:

$$\left| \frac{q}{p} \right|^{\text{th}} = \left| \frac{1 - \epsilon^{\text{th}}}{1 + \epsilon^{\text{th}}} \right| = 0.96 \quad (59)$$

which is in fairly good agreement with the experimental value.

4.4 D-mesons

The other example is the CP -violation in the decay of D meson. The experimental values for CP -violation of $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ as reported by Belle [28] are as follows:

$$\frac{\Delta\Gamma}{2\Gamma} = (0.37 \pm 0.25_{-0.13-0.08}^{+0.07+0.07}), \quad (60)$$

$$\frac{\Delta m}{\Gamma} = (0.81 \pm 0.30_{-0.07-0.16}^{+0.10+0.09}) \quad (61)$$

where $1/\Gamma = \tau$, ($\hbar = 1$) is the mean life time

$$\frac{1}{\Gamma} = \tau = \frac{\tau_{\bar{D}^0} + \tau_{D^0}}{2} = (410.1 \pm 1.5) \times 10^{-3} \text{ ps} \quad (62)$$

The CP -violation parameters are experimentally denoted by $\left(\frac{q}{p}\right)$ and given by:

$$\left|\frac{q}{p}\right|^{\text{exp}} = \left|\frac{1 - \epsilon^{\text{exp}}}{1 + \epsilon^{\text{exp}}}\right| = (0.86_{-0.29-0.03}^{+0.30+0.06}) \quad (63)$$

and

$$\phi^{\text{exp}} = \arg\left(\frac{q}{p}\right)^{\text{exp}} = \arg\left(\frac{1 - \epsilon^{\text{exp}}}{1 + \epsilon^{\text{exp}}}\right) = (-14_{-18-3-4}^{+16+5+2})^\circ. \quad (64)$$

By replacing in the expression (51) we obtain

$$\epsilon^{\text{th}} = (0.077 + 0.035i). \quad (65)$$

Consequently,

$$\left|\frac{q}{p}\right|^{\text{th}} = 0.86, \quad \phi^{\text{th}} = -4.02^\circ. \quad (66)$$

which is in fairly good agreement with the experimental value.

5 Conclusions

As we have shown, if one takes fully account of the subtle distinction between decay rate and integrated survival probability, the CP -violation parameter that we derive from the experiment must be re-estimated, on the basis of our spatial-temporal wave function analogy. We showed how a simple model that we developed in the past [1, 21] provides a correct prediction of the magnitude of the observed CP -violation. We also applied this model to other particle decay data that also reveal CP -violation and discussed the accordance between theoretical predictions and observations in those cases.

It is worth noting that the problem of associating a temporal distribution to a superposition of exponential decay processes is intimately related to the possibility of defining a Time Operator in quantum mechanics, which is also a controversial question. Pauli showed thanks to very simple arguments that if one could find an operator \hat{T} that satisfies canonical commutation rules

$[\hat{T}, \hat{H}] = i\hbar$ with the Hamiltonian operator \hat{H} of a quantum system, then the spectrum of \hat{H} ought to be unbounded by below, which clearly constitutes a physical impossibility. A possible way to answer Pauli's objection is to define a "super" time operator that acts onto density matrices rather than onto pure states. It is not our purpose to investigate this question in the present work, although the results that we derive here were to a large extent inspired by our study of the time operator (this work will be presented in a separate publication). The present approach however does not presuppose the existence of a Time Operator. Neither does it rely on a particular interpretation of the Quantum Theory[29, 30, 31]. In that separate publication (still in preparation), we show that, in the Wigner-Weisskopf approximation, when the energy spectrum extends from $-\infty$ to $+\infty$, the exponential amplitude probability is obtained from a time operator representation of the wave function, where the Time (super)operator is defined as conjugated to the Hamiltonian (super)operator. It is a Fourier transform of a resonance with a complex pole in the energy representation. The idea that underlies the (super)time operator approach is that Pauli's objections are valid in the Hilbert space of pure states, but are not valid in the (super)operator space.

Acknowledgment

T.D. acknowledges support from the ICT Impulse Program of the Brussels Capital Region (Project Cryptasc), the IUAP programme of the Belgian government, the grant V-18, and the Solvay Institutes for Physics and Chemistry. Thanks to Jean-Marie Frere (ULB) and Pascal David (Paris 7) for helpful comments and precious informations.

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